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THERMAL STATE OF COATED SOLIDS IN ASYMMETRIC CYCLIC HEAT EXCHANGE WITH AMBIENT MEDIA

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An analytical solution with improved convergence at the external boundary of the problem of transient heat conduction in a solid–coating system that is cyclically washed by media with different temperatures is given. The solids in question are a plate, a cylinder, and a sphere. It is suggested that the duration of adjacent half-periods, the laws of change in the temperatures of the media in each, and the heat-transfer coefficients may not be the same.

The cyclic thermal interaction of solids with media flowing around them, which is characteristic of internal combustion engines, regenerative heat exchangers, etc., is often accompanied by the deposition on heat-exchange surfaces of substances with thermophysical properties other those of the solids. For example, the packings of regenerative heat exchangers during operation are covered with a layer of carbon, ash, dust, rust, moisture, hoar-frost, etc. Investigation of the influence of the geometric and thermophysical parameters of the coating on the development of temperature fields in solid–coating systems is essential for correct evaluation of the characteristics of the power-generating plant as a whole.

The known solutions of problems of transient heat conduction in layered plates [1] and cylinders [2] do not provide for the cyclicity of heat exchange with the heat-transfer agents that wash them, which, as is shown in [3-5], has a substantial effect on the character and amplitude of temperature fluctuations in the solid in a half-period both for a temperature of the heat-transfer agent that is constant during the half-period [3-4] and for its change with time [5].

In real power-generating plants, in adjacent half-periods the heat-transfer coefficients, the laws of change in the temperatures of the media with time, and the durations of the half-periods often do not coincide with one another, i.e., cyclic heat exchange of solids with heat-transfer agents is asymmetric in time.

The aim of the present work is to refine a mathematical model of cyclic heat exchange of solids with ambient media by taking account of the influence of a coating on the surface of the solids and the asymmetry of the half-periods in time.

Using the agreed-upon assumptions of the absence of thermal resistance on the surface of contact of the solids with the coating and the central symmetry of the temperature field, the boundary-value problem can be represented as:

$$\frac{\partial\Theta_{1,j}(x,\text{Fo})}{\partial\text{Fo}}=\frac{1}{x^{2\nu+1}}\frac{\partial}{\partial x}\left[x^{2\nu+1}\frac{\partial\Theta_{1,j}(x,\text{Fo})}{\partial x}\right]\quad\text{for}\quad 0<x\le 1\,, \tag{1}$$

$$K_a\frac{\partial\Theta_{2,j}(x,\text{Fo})}{\partial\text{Fo}}=\frac{1}{x^{2\nu+1}}\frac{\partial}{\partial x}\left[x^{2\nu+1}\frac{\partial\Theta_{2,j}(x,\text{Fo})}{\partial x}\right]\quad\text{for}\quad 1<x\le 1+h\,; \tag{2}$$

$$\begin{aligned}&\Theta_{1,j}(x,0)=\Theta_{1,j+1}(x,\text{Fo}_{T,j+1})\,,\quad \Theta_{1,j}(x,\text{Fo}_{T,j})=\Theta_{1,j+1}(x,0)\,,\\&\Theta_{2,j}(x,0)=\Theta_{2,j+1}(x,\text{Fo}_{T,j+1})\,,\quad \Theta_{2,j}(x,\text{Fo}_{T,j})=\Theta_{2,j+1}(x,0)\,;\end{aligned} \tag{3}$$

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$$\frac{\partial \Theta_{1,j}(0, \mathrm{Fo})}{\partial x} = 0 ; \tag{4}$$

$$\Theta_{1,j}(1, \mathrm{Fo}) = \Theta_{2,j}(1, \mathrm{Fo}), \quad K_\lambda = \frac{\partial \Theta_{1,j}(1, \mathrm{Fo})}{\partial x} = \frac{\partial \Theta_{2,j}(1, \mathrm{Fo})}{\partial x} ; \tag{5}$$

$$\frac{\partial \Theta_{2,j}(1+h, \mathrm{Fo})}{\partial x} = - K_\lambda \mathrm{Bi}_j \left[\Theta_{2,j}(1+h, \mathrm{Fo}) - \Theta_{f,j}(\mathrm{Fo})\right] . \tag{6}$$

A solution of Eqs. (1) and (2) for boundary conditions (4)-(16) is the series [6]:

$$\begin{aligned} \Theta_{1,j}(x, \mathrm{Fo}) &= \sum_{n=1}^{\infty} x^{2\nu+1} K_1(\mu_{n,j}, x) f(\mu_{n,j}, \mathrm{Fo}) , \\ \Theta_{2,j}(x, \mathrm{Fo}) &= \sum_{n=1}^{\infty} x^{2\nu+1} K_2(\mu_{n,j}, x) f(\mu_{n,j}, \mathrm{Fo}) . \end{aligned} \tag{7}$$

Here $\mu_{n,j}$ are roots of the characteristic equation for the j-th half-period:

$$\mathrm{Bi}_j K_\lambda K_2(\mu_j, 1+h) + \frac{dK_2(\mu_j, 1+h)}{dx} = 0 ;$$

$$K_1(\mu_n, x) = \begin{cases} \cos(\mu_n x) & \text{for } \nu = -1/2 , \\ J_0(\mu_n x) & \text{for } \nu = 0 , \\ \sin(\mu_n x)/x & \text{for } \nu = 1/2 ; \end{cases}$$

$$K_2(\mu_n, x) = \begin{cases} B_1 \cos(\mu_n \sqrt{K_a} x) + B_2 \sin(\mu_n \sqrt{K_a} x) & \text{for } \nu = -1/2 , \\ B_1 J_0(\mu_n \sqrt{K_a} x) + B_2 Y_0(\mu_n \sqrt{K_a} x) & \text{for } \nu = 0 , \\ [B_1 \sin(\mu_n \sqrt{K_a} x) + B_2 \cos(\mu_n \sqrt{K_a} x)]/x & \text{for } \nu = 1/2 ; \end{cases}$$

$$B_1 = \begin{cases} \sqrt{K_a} \cos\mu_n \cos(\mu_n \sqrt{K_a}) + K_\lambda \sin\mu_n \sin(\mu_n \sqrt{K_a}) & \text{for } \nu = -1/2 , \\ -\sqrt{K_a} J_0(\mu_n) Y_1(\mu_n \sqrt{K_a}) + K_\lambda J_1(\mu_n) Y_0(\mu_n \sqrt{K_a}) & \text{for } \nu = 0 , \\ \sqrt{K_a} \sin\mu_n \sin(\mu_n \sqrt{K_a}) + & \\ + \left[K_\lambda \cos\mu_n + (K_\lambda - 1)\sin\mu_n/\mu_n\right] \cos(\mu_n \sqrt{K_a}) & \text{for } \nu = 1/2 ; \end{cases}$$

$$B_2 = \begin{cases} \sqrt{K_a} \cos\mu_n \sin(\mu_n \sqrt{K_a}) - K_\lambda \sin\mu_n \cos(\mu_n \sqrt{K_a}) & \text{for } \nu = -1/2 , \\ \sqrt{K_a} J_0(\mu_n) J_1(\mu_n \sqrt{K_a}) - K_\lambda J_1(\mu_n) J_0(\mu_n \sqrt{K_a}) & \text{for } \nu = 0 , \\ \sqrt{K_a} \sin\mu_n \cos(\mu_n \sqrt{K_a}) - & \\ - \left[K_\lambda \cos\mu_n + (K_\lambda - 1)\sin\mu_n/\mu_n\right] \sin(\mu_n \sqrt{K_a}) & \text{for } \nu = 1/2 ; \end{cases}$$

$$f(\mu_{n,j}, \mathrm{Fo}) = A_{n,j} F_{n,j} + \exp(-\mu_{n,j}^2 \mathrm{Fo})\, \Theta_{L}(\mu_{n,j}, 0) ,$$

$$F_{n,j} = \mu_{n,j}^2 \exp(-\mu_{n,j}^2 \mathrm{Fo}) \int_0^{\mathrm{Fo}} \Theta_{f,j}(\eta) \exp(\mu_{n,j}^2 \eta)\, d\eta ,$$

$$A_{n,j} = \mathrm{Bi}_j K_2(\mu_{n,j}, 1+h)/(\mu_{n,j}^2 S_{n,n}) , \quad S_{n,n} = \int_0^{1+h} x^{2\nu+1} K^2(\mu_{n,j}, x)\, dx ,$$

$$K(\mu_{n,j}, x) = \delta K_1(\mu_{n,j}, x) + (1-\delta)\sqrt{\left(\frac{K_a}{K_\lambda}\right)}\, K_2(\mu_{n,j}, x),$$

$$\delta = \begin{cases} 1 & \text{for } x \le 1, \\ 0 & \text{for } x > 1. \end{cases}$$

The value of $F_{n,j}$ accounts for the influence of the law of change in the medium's temperature with time $\Theta_{f,j}(\mathrm{Fo})$ on the development of the temperature fields in the solid-coating system; its form is dependent on the form of the function $\Theta_{f,j}(\mathrm{Fo})$. For example, for an exponential law

$$\Theta_{f,j}(\mathrm{Fo}) = [2(j-1) + b_j]\,[1 - \exp(-\beta_j \mathrm{Fo})], \tag{8}$$

where b_j and β_j are constants (for symmetry of the half-periods $b_j = -b_{j+1} = b$, $\beta_j = -\beta_{j+1} = \beta$), the function $F_{n,j}$ becomes equal to:

$$F_{n,j} = \left[2(j-1) + \beta_j\right]\left[1 - \exp(-\mu_{n,j}^2 \mathrm{Fo}) - \mu_{n,j}^2 \frac{\exp(-\beta_j \mathrm{Fo}) - \exp(-\mu_{n,j}^2 \mathrm{Fo})}{\mu_{n,j}^2 - \beta_j}\right].$$

Similarly we can obtain expressions for $F_{n,j}$ for other dependences $\Theta_{f,j}(\mathrm{Fo})$, whose class is mathematically, unlimited.

The function of $\Theta_L(\mu_{n,j}, 0)$ is a representation of the initial field in the j-th half-period and is found from initial conditions (3):

$$\Theta_L(\mu_{n,j}, 0) = \Theta_L^{(1)}(\mu_{n,j}, 0) + \varphi_{n,j} \sum_{m=1}^{\infty} \frac{S_{n,m}}{S_{m,m}} \exp(-\mu_{m,j+1}^2 \mathrm{Fo}_{T,j+1}) \sum_{\substack{l=1 \\ l \ne n}}^{\infty} S_{m,l}\Theta_L^{(1)}(\mu_{l,j}, 0) \exp(-\mu_{l,j}^2 \mathrm{Fo}_{T,j}),$$

where

$$\Theta_L^{(1)}(\mu_{n,j}, 0) = \varphi_{n,j} \sum_{m=1}^{\infty} S_{n,m}\left[A_{m,j+1}F_{m,j+1} + \frac{\exp(-\mu_{m,j+1}^2 \mathrm{Fo}_{T,j+1})}{S_{m,m}} \sum_{l=1}^{\infty} S_{m,l}A_{l,j}F_{l,j}\right];$$

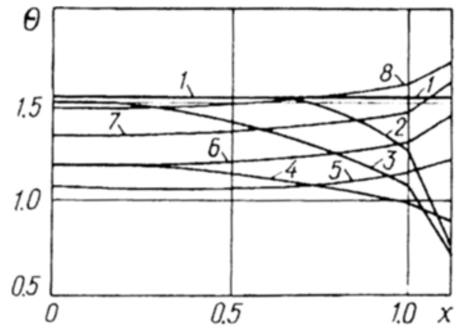
$$\varphi_{n,j} = \left[S_{n,n} - \exp(-\mu_{n,j}^2 \mathrm{Fo}) \sum_{m=1}^{\infty} \frac{S_{n,m}}{S_{m,m}} \exp(-\mu_{m,j+1}^2 \mathrm{Fo}_{T,j+1})\right]^{-1};$$

$$S_{n,m} = \int_0^{1+h} x^{2\nu+1} K(\mu_{n,j}, x)\, K(\mu_{m,j+1}, x)\, dx;$$

$$S_{m,m} = \int_0^{1+h} x^{2\nu+1} K^2(\mu_{m,j+1}, x)\, dx;$$

$$A_{m,j+1} = \mathrm{Bi}_{j+1} K_2(\mu_{m,j+1}, 1+h)/(\mu_{m,j+1}^2 S_{m,m}).$$

Solution (7) of boundary-value problem (1)-(6) has a drawback that is inherent in finite integral transforms: the nonuniform convergence of series (7) at the external boundary of the system, i.e., for $x = 1 + h$, for inhomogeneous boundary conditions. The covergence of the solution obtained can be improved by the method of G. A. Grinberg as applied to the problems of heat conduction considered in [7, 8]. Applying this method to the given boundary-value problem permits a final solution with improved convergence at the external boundary of the coating:



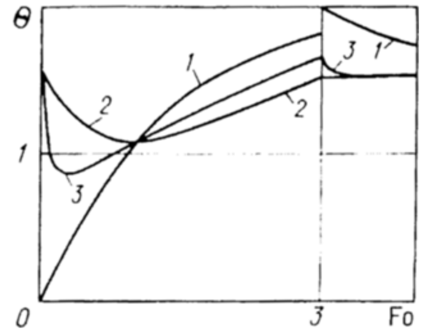


Fig. 1. Temperature fields in plate and coating: 1) Fo = 0; 2) 0.03; 3) 0.139; 4) 0.6; 5) 1.2; 6) 1.8; 7) 2.4; 8) 3.

Fig. 2. Temperatures of media and the average temperatures of plate and coating vs time: 1) temperature of media; 2) of plate; 3) of coating.

$$\Theta_{1,j}(x, \text{Fo}) = \Theta_{f,j}(\text{Fo}) + \sum_{n=1}^{\infty} G(\mu_{n,j}, \text{Fo})\, x^{2\nu+1} K_1(\mu_{n,j}, x)\,,$$
$$\Theta_{2,j}(x, \text{Fo}) = \Theta_{f,j}(\text{Fo}) + \sum_{n=1}^{\infty} G(\mu_{n,j}, \text{Fo})\, x^{2\nu+1} K_2(\mu_{n,j}, x)\,, \tag{9}$$

where

$$G(\mu_{n,j}, \text{Fo}) = f(\mu_{n,j}, \text{Fo}) - A_{n,j}\Theta_{f,j}(\text{Fo})\,.$$

Integration of expressions (9) with respect to x from the solid to the coating yields relations for the averaged temperatures:

$$\overline{\Theta}_{1,j}(\text{Fo}) = \Theta_{f,j}(\text{Fo}) + \sum_{n=1}^{\infty} G(\mu_{n,j}, \text{Fo}) \int_0^1 x^{2\nu+1} K_1(\mu_{n,j}, x)\, dx\,,$$
$$\overline{\Theta}_{2,j}(\text{Fo}) = \Theta_{f,j}(\text{Fo}) + \sum_{n=1}^{\infty} G(\mu_{n,j}, \text{Fo}) \int_1^{1+h} x^{2\nu+1} K_2(\mu_{n,j}, x)\, dx\,. \tag{10}$$

As an example that illustrates some possibilities of the solution obtained, we consider the cyclic heat exchange of a steel plate covered by an oil layer with two media whose temperatures change according to law (8). The similarity numbers and the constants that characterize the system in question, the duration of the half-periods, and the boundary conditions are as follows: $h = 0.001$; $K_\lambda = 397$; $K_a = 208$; $\text{Bi}_1 = 2$; $\text{Fo}_{T,1} = 3$; $b_1 = 2$; $\beta_1 = 0.8$; $\text{Bi}_2 = 0$; $\text{Fo}_{T,2} = 1$, $b_2 = -0.4$; $\beta_2 = 1$. The results of the calculations of the temperature fields by relations (9) are shown in Fig. 1. Despite the relatively large value of Bi_1, good convergence of the solution on the outer surface of the coating is attained for a comparatively small number of the terms in series (9), which is equal to 10. The figure shows that in the first half-period ($0 \le \text{Fo} \le \text{Fo}_{T,1}$), the thermal state of the system tends to approach the ambient tempearture, and the temperature fields are nonuniform in the supporting wall and especially in the coating. On the contact surface, the plate and wall temperatures are equal but the temperature gradients are different, depending on the value of the constant K_λ (in Fig. 1, this difference is not so evident, because the scale of the coating is larger by a factor at 100 than the scale of the supporting wall). In the second half-period, the plate–coating system is not involved in heat exchange and the temperature field levels off, becoming uniform at the end of the half-period. The dynamics of change in the average temperatures of the supporting plate and the coating with time calculated by Eqs. (10) is shown in Fig. 2. It can be seen that the temperature fluctuations in the plate are due only to its heat exchange with the medium in the first half-period.

The mathematical model considered here and the solution obtained will be useful in refining temperature fields and heat-transfer coefficients in different thermal power-generating plants.

NOTATION

$\nu = -1/2, 0, 1/2$, respectively, for plate, cylinder, and sphere; $j = 1, 2$, half-period number; $\Theta_{1,j}(x, \mathrm{Fo})$, relative excess temperature of solid in j-th half-period at point x, $0 < x \le 1$; $\Theta_{2,j}(x, \mathrm{Fo})$, the same of coating, $1 < x \le 1 + h$; h, relative thickness of coating; $\mathrm{Fo} = a_1\tau/l_1^2$, Fourier number; l_1, characteristic size of solid, m; τ, time from start of current j-th half-period, sec; $\mathrm{Fo}_{T,j} = a_1 T_j/l_1^2$, limiting Fourier number for j-th half-period; T_j, duration of j-th half-period, sec; $K_\lambda = \lambda_1/\lambda_2$; $K_a = a_1/a_2$; λ_1 and λ_2, thermal conductivity coefficients of solid and coating, W/(m·K); a_1 and a_2, thermal diffusivity coefficients of solid and coating, $\mathrm{m^2/sec}$; $\mathrm{Bi}_j = \alpha_j l_1/\lambda_1$, Biot number of j-th half-period; α_j, heat-transfer coefficient in j-th half-period, $\mathrm{W/(m^2 \cdot K)}$; $\Theta_f(\mathrm{Fo})$, relative excess temperature of medium; $\overline{\Theta}_{1,j}(\mathrm{Fo})$ and $\overline{\Theta}_2(\mathrm{Fo})$, temperatures of solid and coating in j-th half-period averaged over thickness; $J_r(z)$ and $Y_r(z)$, Bessel functions of first and second kinds of the r-th order.

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